Law of total expectation

Theorem 1:

Let:

be a random variable.

be a random variable defined on same probability space.

Statement:

Then we have that:

Proof:

By definition of expected value, we can have that

Since for all y belongs to Y, is a not a variable about X, we can put

inside the sigma, getting:

With law of conditional probability, this will become:

With law of marginal probability, this will become:

Hence proved.

Ref:

[Law of total expectation | The Book of Statistical Proofs (statproofbook.github.io)](https://statproofbook.github.io/P/mean-tot)

[Law of total expectation - Wikipedia](https://en.wikipedia.org/wiki/Law_of_total_expectation#Proof_in_the_finite_and_countable_cases)